Correspondence

Discussion of Line Width and Gyromagnetic Ratio*

A considerable amount of work has been reported on the measurement and interpretation of line width and gyromagnetic ratio of microwave ferrite materials. Most frequently, measurements are made on a small sphere placed in a resonant cavity.[1-4] It is the purpose of this letter to summarize some of the significant results of this work, particularly from the point of view of an engineer using ferrites.

Ferromagnetic Resonance

Ferrites at microwave frequencies have losses that depend on the material, its geometry, the frequency, and the magnetic biasing field. Consider first that the ferrite is unbounded. For a fixed frequency, the curve of absorption of energy by the ferrite vs biasing field exhibits a resonance, and is similar to the curve of current vs capacitance of a series LCR circuit. Resonance occurs at a field H given by

$$H = \omega/\gamma \qquad (1)$$

where ω is the angular test frequency and γ is the gyromagnetic ratio. This is a ferromagnetic resonance at which the susceptibility (ratio of RF magnetization to RF magnetic field) has a resonant rise.

Line width is a measure of the sharpness of the resonant rise of absorption: the smaller the line width, the sharper the resonant rise. In this way, line width is analogous to the dissipation factor of an LCR circuit. Line width is defined as the separation of the two magnetic-field values at which the loss component of susceptibility is one half its maximum value.

Nature of Absorption Resonance in the Case of a Sphere

In practice, measurements of line width and gyromagnetic ratio are made using a small sphere in a resonant cavity. The maximum energy absorbed by the cavity, and hence the greatest decrease in cavity Q, will occur at an externally applied magnetic field determined from (1). However, this is not a case of ferromagnetic resonance, since that is determined by the internal biasing field, and the internal biasing field in the sphere is smaller than the applied field because of demagnetizing effects. Hogan shows that the resonant rise in absorption is due to a resonant buildup of the RF fields at the bias determined by (1).[5] However, the susceptibility is relatively small, much smaller than the susceptibility at ferromagnetic resonance. Susceptibility in this letter will always be taken to mean the internal or intrinsic susceptibility; *i.e.*, the ratio of magnetization to internal RF fields.

It is instructive to consider the formula for the change in the resonant frequency of the cavity as a function of the intrinsic permeability, μ_i, of the sample material. (Circular polarization is assumed.) $\mu_i = 1+\mu+k = 1+\mu'+k'-j(\mu''+k'')$ where μ and k are the diagonal and off-diagonal terms in the susceptibility tensor.[2,6]

$$\frac{\Delta F}{F} = A\,\frac{\mu_i - 1}{\mu_i + 2}\,. \qquad (2)$$

The frequency F is complex, and A is a geometric constant. This formula shows that the greatest change in complex frequency occurs when μ_i approaches -2. Also, note that ΔF is relatively small for very large μ_i.

From the above discussion, it might appear that the line width as measured in a sphere is only indirectly related to the intrinsic line width that would be measured in an infinite medium. However, if we assume the simple mechanism of loss implied in the Landau-Lifschitz formulation and disregard the effects of porosity and anisotropy, the line width as measured in the sphere will be identical to the line width that would be measured in an infinite medium.[7]

Nature of Absorption in the Case of a Rod, Disk

As in the case of the sphere, the resonance in the absorption of a long thin rod will be due to a resonant buildup of the RF fields in the sample rather than to a ferromagnetic resonance. Also, under the assumptions listed above, the line width in this case will be the same as in the infinite medium.[7]

In the case of a thin disk, the resonance of the absorption will be a true ferromagnetic resonance. The internal biasing field at resonance will be the same as in the case of the infinite medium, and the line width will be the same as in the infinite medium even if porosity and anisotropy are taken into account. However, despite this advantage for thin disks, measurements are usually made on spheres principally because of the fact that the spheres are the only true ellipsoids that can readily be prepared.

Effects of Porosity and Anisotropy

Porosity and anisotropy cause local variations in the magnetic bias within the material, and this causes a broadening of the line width. They also cause the measured line width to have a different shape from the simple symmetrical shape predicted by the *L-L* formulation, since the *L-L* formulation disregards porosity and anisotropy. Since, for resonance at a given frequency, the magnitude of the internal bias field in the case of the sphere, rod, and disk are different one from the other, and since the effects of anisotropy and porosity depend to some extent on the magnitude of the bias field, the measured line width will depend on the sample shape.[8]

It is important to note that magnetic losses far off resonance are often much lower than the loss which would be predicted on the basis of the measured line width. This has been observed experimentally by Rowen and Von Aulock.[2] Far off resonance, the losses are approximately those that would have been predicted on the basis of a line width that would exist if there were no broadening caused by anisotropy and porosity. The reason for this may be understood by the radio engineer from the following circuit analogy. Consider two tuned amplifiers. In the one, the tuned circuits in each of the stages are identical. In the second amplifier, the tuned circuits are similar to those of the first, but each circuit is tuned to a slightly different frequency. The second amplifier will have a much broader resonance than the first. However, far off resonance, the response of both amplifiers will be close.

Surface Roughness, Spin Waves

For materials with very narrow intrinsic line widths, such as single crystals of yttrium iron garnet, the surface roughness of the sphere being tested can have a predominant role in determining line width. Also, for such materials, losses caused by spin wave coupling become important.[9] Since the losses due to the spin wave coupling depend in part on the demagnetizing factors, the shape of the specimen may appreciably affect the line width.

Sample Size of the Sphere

When we use perturbation techniques, the sample must be sufficiently small that the external RF field it sees is essentially

* Received by the PGMTT, December 3, 1959.

[1] J. O. Artman and P. E. Tannenwald, "Measurement of susceptibility tensor in ferrites," *J. Appl. Phys.*, vol. 26, pp. 1124–1132; September, 1955.

[2] J. H. Rowen and W. Von Aulock, "Measurement of the dielectric and magnetic properties of ferromagnetic materials at microwave frequencies," *Bell Sys. Tech. J.*, vol. 36, pp. 427–448; March, 1957.

[3] E. G. Spencer, R. C. LeCraw, and F. Reggia, "Measurement of the microwave dielectric constants and tensor permeabilities of ferrite spheres," Proc. IRE, vol. 44, pp. 790–800; June, 1956.

[4] R. A. Waldron, "Resonant cavity methods of measuring ferrite properties," *Brit. J. Appl. Phys.*, vol. 9, pp. 439–442; November, 1958.

[5] C. L. Hogan, "Elements of nonreciprocal microwave devices," Proc. IRE, vol. 44, pp. 1345–1368; October, 1956. See p. 1353.

[6] R. A. Waldron, "Ferrites in resonant cavities," *Brit. J. Appl. Phys.*, vol. 7, p. 114; March, 1956.

[7] A. D. Berk, "Dependence of the ferromagnetic resonance line width on the shape of the specimen," *J. Appl. Phys.*, vol. 28, pp. 190–192; February, 1957.

[8] E. G. Spencer, L. A. Ault, and R. C. LeCraw, "Intrinsic tensor permeabilities of ferrite rods, spheres, disks," Proc. IRE, vol. 44, pp. 1311–1317; October, 1956.

[9] C. R. Buffler, "Ferrimagnetic resonance near upper limit of the spin wave manifold," *J. Appl. Phys.*, suppl. to vol. 30, pp. 172S–175S; April, 1959.

uniform. Also, the sample must be sufficiently small that there are no "retardation" or propagation effects within the sample. The propagation constant within the sample can be calculated from its dielectric constant and taking its permeability as -2. [See (2).] Tompkins and Spencer have derived a formula to take sample size into account.[10] Their formula when expanded in a Taylor series, becomes

$$\frac{\Delta F}{F} = A \frac{(\mu_i - 1)\left(1 - \frac{X^2}{10}\right)}{(\mu_i + 2) - X^2\left(\frac{\mu_i - 1}{10} + \frac{1}{2}\right)} \cdot (3)$$

$X = 2\pi r/\lambda_0 \sqrt{2\epsilon'}$ where r is the radius of the sample, ϵ' is the real part of the dielectric constant of the ferrite (approximately 10), amd λ_0 is the free space wavelength. According to this equation, the error in gyromagnetic ratio measured at 3000 megacycles for a sample diameter of 0.240 inch, or at 9000 megacycles for a sample diameter of 0.080 inch, will be about two per cent. The error in line width is comparable.

Spencer *et al.* give experimental data to show that the line width of $R-1$ measured at X band is independent of sample size for diameters ranging from 25 to 60 mils.[11] Stinson shows that the line width of polycrystalline YIG measured at X band is independent of sample size for diameters ranging from 40 to 90 mils.[12] Stinson attributes the independence of sample diameter to his use of a cross-guide coupler instead of a resonant cavity, and refers to an article by Artman to show that sample size has a strong effect on measurement of line width when a cavity is used.[13] However, as noted above, Tompkins and Spencer have derived an equation for measurement in a cavity which shows only a very small dependence on diameter for the range of diameters covered by Stinson. Tompkins and Spencer discuss the discrepancy between their equation and that of Artman, and this writer believes that Tompkins and Spencer are correct.

The writer appreciates helpful comments received from Dr. R. C. LeCraw of Bell Telephone.

I. BADY
U. S. Army Signal Res. and Dev. Lab.
Fort Monmouth, N. J.

[10] J. E. Tompkins and E. G. Spencer, "Retardation effects caused by ferrite sample size on the frequency shift of a resonant cavity," *J. Appl. Phys.*, vol. 28, pp. 969–974; September, 1957.

[11] E. G. Spencer, R. C. LeCraw, and L. A. Ault, "Note on cavity perturbation theory," *J. Appl. Phys.*, vol. 28, pp. 130–132; January, 1957.

[12] D. C. Stinson, "Experimental techniques in measuring ferrite line widths with a cross-guide coupler," 1958 WESCON CONVENTION RECORD, pt. 1, pp. 147–150.

[13] J. O. Artman, "Effects on the microwave properties of ferrite rods, discs, and spheres," *J. Appl. Phys*, vol. 28, pp. 92–98; January, 1957.

Plotting Impedances with Negative Resistive Components*

The plotting of impedances with negative resistive components on some sort of inverted Smith chart is becoming more common.[1] This note suggests standardizing on a particular form, for psychological reasons. The suggested form is represented by "Γ" $= -1/\Gamma$, where Γ is the actual complex reflection coefficient, and "Γ" is the value plotted on the chart. The corresponding impedance relation is "Z/Z_0" $= -Z_0/Z$. The advantages claimed for this particular form are:

1) The transformation is analytic as opposed to the one mentioned by Stock and Kaplan.[1]

2) If both negative and positive resistances are being plotted on two Smith charts, the result, as shown in Fig. 1, looks like the representation of the world on the covers of some atlases. It fits well with the concept of projection on the unit sphere.

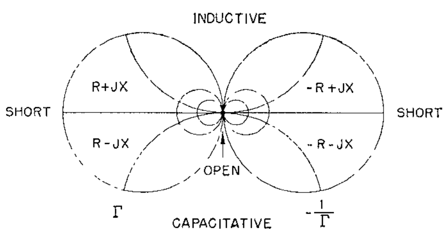

Fig. 1—Double SMITH-HTIMS chart.

3) It follows from 1) that if impedance is being plotted as a function of real frequency, then points of stability or instability as indicated by complex frequency will fall on the same side of the curve on both charts.

A possible disadvantage is the opposite sense of rotation of the two charts for transmission-line calculations, but this seems natural and easy to remember for two circles in "contact."

R. L. KYHL
Electrical Eng. Dept. and
Res. Lab. of Electronics
Mass. Inst. Tech.
Cambridge, Mass.

* Received by the PGMTT, January 18, 1960.

[1] D. J. R. Stock and L. J. Kaplan, "The representation of impedances with negative real parts in the projective chart," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, p. 475, October, 1959; L. J. Kaplan and D. J. R. Stock, "An extension of the reflection coefficient chart to include active networks," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 298–299; April, 1959. R. M. Steere, "Novel applications of the Smith chart," *Microwave J.*, vol. 3, pp. 97–100; March, 1960.

Comments on "The Design of Ridged Waveguide"*

An article by Hopfer,[1] which appeared in 1955, takes into account the step discontinuity susceptance in the computation of the cutoff frequencies of ridged-waveguide. Cutoff frequencies are computed utilizing the transverse resonance method. Values of the normalized step susceptance that were used in computing the cutoff frequencies were taken from published data in the Waveguide Handbook.[2]

It seems that this procedure for determining the step susceptance is questionable. The transverse resonance method as applicable to ridged waveguide entails computing the circuit parameters of parallel plane transmission lines. Consequently, the step discontinuity susceptance should be computed as a step in a parallel plane transmission line[3,4] rather than as a step in rectangular waveguide.

RICHARD M. KURZROK
RCA Surface Communications
Systems Laboratory
75 Varick Street
New York 13, N. Y.

* Received by the PGMTT, January 18, 1960.

[1] S. Hopfer, "The design of ridged waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 20–29; October, 1955.

[2] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10, pp. 399–402; 1951.

[3] S. B. Cohn, "Properties of ridge waveguide," PROC. IRE, vol. 35, pp. 783–788; August, 1947.

[4] J. R. Whinnery and H. W. Jamieson, "Equivalent circuits for discontinuities in transmission lines," PROC. IRE, vol. 32, pp. 98–116; February, 1944.